

## Supersymmetry of a nonstationary Pauli equation

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1998 J. Phys. A: Math. Gen. 31 1859

(<http://iopscience.iop.org/0305-4470/31/7/017>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 171.66.16.104

The article was downloaded on 02/06/2010 at 07:23

Please note that [terms and conditions apply](#).

# Supersymmetry of a nonstationary Pauli equation

V M Tkachuk†

Ivan Franko Lviv State University, Chair of Theoretical Physics, 12 Drahomanov Str., Lviv  
UA-290005, Ukraine

Received 4 November 1997

**Abstract.** The supersymmetry of the electron in both the nonstationary magnetic and electric fields in a two-dimensional case is studied. The supercharges which are the integrals of motion and their algebra are established. Using the obtained algebra the solutions of nonstationary Pauli equation are generated.

## 1. Introduction

Supersymmetric quantum mechanics introduced in [1, 2] currently receives much attention (see also surveys [3–5]). One of the quantum-mechanical problems where supersymmetry (SUSY) is the physical symmetry is the motion of the electron in a stationary two-dimensional magnetic fields and in some three-dimensional fields [6–10] (see also [3–5] and references therein).

The problem of introducing SUSY in the case of the electron motion in the nonstationary magnetic field was investigated for the first time in [11]. The case of a time-varying uniform magnetic field with the fixed direction was examined and it was shown that the group-theoretical analysis can provide a clue to the supersymmetric factorization of the time-dependent Pauli equation and to obtaining eigensolutions. Note also that recently the SUSY of a one-dimensional time-dependent Schrödinger equation was established and with the help of the time-dependent Darboux transformation, new exactly solvable time-dependent potentials were generated [12].

The aim of this paper is to establish the SUSY of the electron in both the nonstationary magnetic and electric fields. The supercharges are obtained as integrals of motion. They are the straightforward generalization of the well known supercharges in the stationary case.

## 2. SUSY of the Pauli equation in a two-dimensional case

In this paper we consider the electron motion in nonstationary ‘two-dimensional’ electromagnetic field. The equation of motion in this case is the nonstationary Pauli equation

$$i\frac{\partial\psi}{\partial t} = H_p\psi \quad (1)$$

† E-mail address: tkachuk@ktf.franko.lviv.ua

where

$$\begin{aligned} H_p &= \frac{1}{2}(\pi_x^2 + \pi_y^2 + p_z^2 - eB\sigma_z) \\ \pi_\alpha &= p_\alpha - eA_\alpha \quad p_\alpha = -i\frac{\partial}{\partial x_\alpha} \\ A_x &= A_x(x, y, t) \quad A_y = A_y(x, y, t) \quad A_z = 0 \end{aligned} \quad (2)$$

are the components of the vector potential,

$$B = B_z(z, y, t) = \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}$$

is the magnetic field parallel to the  $z$ -axis.

It is obvious that  $p_z$  is the integral of motion. Therefore, the solution of equation (1) can be written in the following form

$$\psi(x, y, z, t) = e^{-ik^2t/2+ikz}\psi(x, y, t)$$

where  $k$  is the value of momentum along  $z$ -axis. Then  $\psi(x, y, t)$  satisfies the Pauli equation (1) with a two-dimensional Hamiltonian

$$H = \frac{1}{2}(\pi_x^2 + \pi_y^2 - eB\sigma_z). \quad (3)$$

Further, we shall only deal with the two-dimensional case.

Let us first take a look at the SUSY formulation of the two-dimensional Pauli equation in the case of the stationary magnetic field [6–9]. Then the solution of the nonstationary equation can be written in the form of  $\psi(x, y, t) = e^{-iEt}\psi(x, y)$  and for  $\psi(x, y)$  we obtain the stationary Pauli equation

$$H\psi(x, y) = E\psi(x, y). \quad (4)$$

The two-dimensional Pauli Hamiltonian can be written as follows

$$H = \begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \pi_- \pi_+ & 0 \\ 0 & \pi_+ \pi_- \end{pmatrix} \quad (5)$$

or in supersymmetrical form

$$H = \{Q_+, Q_-\}. \quad (6)$$

The supercharges  $Q_+$ ,  $Q_-$  read

$$\begin{aligned} Q_+ &= \frac{\pi_- \sigma_+}{\sqrt{2}} \\ Q_- &= \frac{\pi_+ \sigma_-}{\sqrt{2}} \end{aligned} \quad (7)$$

where

$$\begin{aligned} \pi_\pm &= \pi_x \pm i\pi_y \\ \sigma_\pm &= \frac{\sigma_x \pm i\sigma_y}{2}. \end{aligned}$$

It is convenient to introduce the complex variable  $z = x + iy$ . Then

$$\begin{aligned} \pi_+ &= -2i\frac{\partial}{\partial z^*} - eA(z, z^*) \\ \pi_- &= -2i\frac{\partial}{\partial z} - eA^*(z, z^*) \\ A(z, z^*) &= A_x + iA_y. \end{aligned} \quad (8)$$

The introduced supercharges (7) also satisfy the following relations

$$\begin{aligned} (Q_{\pm})^2 &= 0 \\ [Q_{\pm}, H] &= 0. \end{aligned} \quad (9)$$

These relations together with (6) define the SUSY algebra, which explains the two-fold degeneracy of the nonzero energy levels of the electron in the two-dimensional inhomogeneous magnetic field  $B(x, y)$ .

Note that the supercharges  $Q_{\pm}$  commute with the Hamiltonian and in the case of the stationary magnetic field they are the integrals of motion. In the case of the nonstationary magnetic field, all relations (5)–(9) are also true but  $Q_{\pm}$  are not the integrals of motion. In the next section we shall establish the supercharges  $\tilde{Q}_{\pm}$  which are the integrals of the electron motion in the nonstationary electromagnetic field.

### 3. Supercharges in a nonstationary case

The supercharges which are the integrals of motion must satisfy the equation

$$i \frac{\partial \tilde{Q}_{\pm}}{\partial t} + [\tilde{Q}_{\pm}, H] = 0. \quad (10)$$

This equation was used in [13] for the calculation of the supercharges in the case of the nonstationary magnetic field and some results of paper [11] concerning SUSY were reproduced in more simple way. In this paper we consider the case of the nonstationary axially symmetric electromagnetic field with the vector potential

$$\begin{aligned} A_x &= -\frac{1}{2}B(t)y + \frac{1}{2}D(t)x \\ A_y &= \frac{1}{2}B(t)x + \frac{1}{2}D(t)y. \end{aligned} \quad (11)$$

Then the magnetic field is

$$B_x = B_y = 0 \quad B_z = B(t). \quad (12)$$

The components of the electric field consist of two parts

$$\begin{aligned} E_x &= \frac{1}{2} \frac{\partial B(t)}{\partial t} y - \frac{1}{2} \frac{\partial D(t)}{\partial t} x \\ E_y &= -\frac{1}{2} \frac{\partial B(t)}{\partial t} x - \frac{1}{2} \frac{\partial D(t)}{\partial t} y \end{aligned} \quad (13)$$

where the first term is the solenoidal field connected with the time-varying of the magnetic field, the second term is a potential electric field of the nonstationary harmonic oscillator. Thus, the system considered is an isotropic two-dimensional nonstationary harmonic oscillator in the time-varying uniform magnetic field. Note that the case  $D(t) = 0$  corresponds to the one considered in [11, 13].

In complex variables

$$A(z, z^*, t) = \frac{1}{2}a(t)z \quad (14)$$

where

$$a(t) = D(t) + iB(t) \quad (15)$$

and then

$$\begin{aligned} \pi_- &= -2i \frac{\partial}{\partial z} - \frac{ea^*(t)}{2} z^* \\ \pi_+ &= -2i \frac{\partial}{\partial z^*} - \frac{ea(t)}{2} z. \end{aligned} \quad (16)$$

The solution of equation (10)  $\tilde{Q}_{\pm}$  can be written in a form similar to (7)

$$\begin{aligned}\tilde{Q}_+ &= \frac{\tilde{\pi}_-\sigma_+}{\sqrt{2}} \\ \tilde{Q}_- &= \frac{\tilde{\pi}_+\sigma_-}{\sqrt{2}}\end{aligned}\quad (17)$$

where

$$\begin{aligned}\tilde{\pi}_- &= -f_1(t)2i\frac{\partial}{\partial z} - f_2(t)\frac{ea^*}{2}z^* \\ \tilde{\pi}_+ &= -f_1^*(t)2i\frac{\partial}{\partial z^*} - f_2^*(t)\frac{ea}{2}z\end{aligned}\quad (18)$$

$f_1(t)$  and  $f_2(t)$  are the unknown functions that will be calculated further. For this purpose the following commutation relations are useful

$$\begin{aligned}[\pi_-, \pi_+] &= ie(a - a^*) = -2eB \\ [\tilde{\pi}_-, \tilde{\pi}_+] &= ie(f_1f_2^*a + f_1^*f_2a^*) \\ [\tilde{\pi}_-, \pi_+] &= ie(f_1a + f_2a^*) \\ [\pi_-, \tilde{\pi}_+] &= -ie(f_1^*a^* + f_2^*a) \\ [\pi_-, \tilde{\pi}_-] &= [\pi_+, \tilde{\pi}_+] = 0.\end{aligned}\quad (19)$$

Substituting  $\tilde{Q}_+$  from (17) into (10) we obtain the equation for  $\tilde{\pi}_-$

$$i\frac{\partial\tilde{\pi}_-}{\partial t} + \tilde{\pi}_-H_- - H_+\tilde{\pi}_- = 0.\quad (20)$$

Before considering the equation for  $f_1$  and  $f_2$  note that  $\tilde{\pi}_-$  is in a simple way related with the integral of motion. Indeed, using the fact that

$$H_+ - H_- = [\pi_-, \pi_+] = -2eB(t)$$

equation (20) can be written in the following form

$$i\frac{\partial\tilde{\pi}_-}{\partial t} + [\tilde{\pi}_-, H_-] + 2eB(t)\tilde{\pi}_- = 0.\quad (21)$$

Let

$$\tilde{\pi}_- = e^{i2\Omega(t)}I\quad (22)$$

where  $\Omega(t) = e\int_0^t B(t) dt$ . Then  $I$  satisfies the equation

$$i\frac{\partial I}{\partial t} + [I, H_-] = 0.\quad (23)$$

Thus,  $I$  is the integral of motion of the Hamiltonian  $H_-$ . It is obvious that  $I$  is also the integral of motion of  $H_+$  and thus the integral of motion of the total Hamiltonian  $H$ . It is also worth noting that

$$\tilde{\pi}_+ = e^{-i2\Omega(t)}I^+\quad (24)$$

where  $I^+$  is the integral of motion conjugated to  $I$ . The question about the integrals of motion of the nonstationary quantum-mechanical problems was studied in [14].

Now, let us consider the equations for  $f_1$  and  $f_2$ . Using the commutation relations (19) and substituting the explicit expression for  $\tilde{\pi}_-$  (18) into (20) or (21) we obtain a set of equations for  $f_1(t)$  and  $f_2(t)$

$$\begin{aligned} \frac{\partial f_1}{\partial t} + ea^*(t)(f_1 - f_2) &= 0 \\ \frac{\partial(f_2a^*(t))}{\partial t} + ea^*(t)a(t)(f_1 - f_2) &= 0. \end{aligned} \tag{25}$$

The solution of this set of equations can be written as follows

$$\begin{aligned} f_1 &= fe^{i\Omega(t)} \\ ef_2a^* &= (eD(t)f + \frac{\partial f}{\partial t})e^{i\Omega(t)} \end{aligned} \tag{26}$$

where  $f$  satisfies the second-order differential equation

$$\frac{\partial^2 f}{\partial t^2} = - \left( (eB(t))^2 + e \frac{\partial D(t)}{\partial t} \right) f. \tag{27}$$

Thus, in our case the problem of constructing the supercharges  $\tilde{Q}_\pm$  leads to equation (27). Solving this equation and using (26) we obtain  $f_1, f_2$  and thus supercharges (17)  $\tilde{Q}_\pm$ , where

$$\begin{aligned} \tilde{\pi}_- &= e^{i\Omega(t)} \left( -f2i \frac{\partial}{\partial z} - \left( \frac{\partial f}{\partial t} + eD(t)f \right) \frac{1}{2}z^* \right) \\ \tilde{\pi}_+ &= e^{-i\Omega(t)} \left( -f^*2i \frac{\partial}{\partial z^*} - \left( \frac{\partial f^*}{\partial t} + eD(t)f^* \right) \frac{1}{2}z \right). \end{aligned} \tag{28}$$

To conclude this section let us consider the stationary magnetic and electric fields

$$\begin{aligned} B(t) &= B = \text{constant} \\ D(t) &= Dt \quad D = \text{constant}. \end{aligned}$$

Equation (27) in this case can be easily solved

$$f = c_1e^{-i\omega t} + c_2e^{i\omega t} \tag{29}$$

where  $\omega = \sqrt{(eB)^2 + eD}$ ,  $c_1, c_2$  are arbitrary constants.

We have two linearly independent solutions. Choosing  $c_1 = 1, c_2 = 0$  gives the following supercharges

$$\begin{aligned} \tilde{Q}_+ &= \frac{1}{\sqrt{2}}e^{i(\omega_0-\omega)t} \left( -2i \frac{\partial}{\partial z} - (eDt - i\omega) \frac{1}{2}z^* \right) \sigma_+ \\ \tilde{Q}_- &= \frac{1}{\sqrt{2}}e^{-i(\omega_0-\omega)t} \left( -2i \frac{\partial}{\partial z^*} - (eDt + i\omega) \frac{1}{2}z \right) \sigma_- \end{aligned} \tag{30}$$

where  $\omega_0 = eB$ . The second linearly independent solution  $c_1 = 0, c_2 = 1$  gives

$$\begin{aligned} \tilde{Q}_+ &= \frac{1}{\sqrt{2}}e^{i(\omega_0+\omega)t} \left( -2i \frac{\partial}{\partial z} - (eDt + i\omega) \frac{1}{2}z^* \right) \sigma_+ \\ \tilde{Q}_- &= \frac{1}{\sqrt{2}}e^{-i(\omega_0+\omega)t} \left( -2i \frac{\partial}{\partial z^*} - (eDt - i\omega) \frac{1}{2}z \right) \sigma_- \end{aligned} \tag{31}$$

Note that even in the constant electromagnetic field supercharges (30) and (31) depend on the time  $t$ . When the electric field is equal to zero  $D = 0$  the supercharges (30) result in

the well known time-independent supercharges (7). Supercharges (31) in the case of  $D = 0$  give new supercharges which are connected with (7) in the following way

$$\begin{aligned}\tilde{Q}_+ &= e^{i2\omega_0 t} Q_+(-B) \\ \tilde{Q}_- &= e^{-i2\omega_0 t} Q_-(-B)\end{aligned}\quad (32)$$

where  $Q_{\pm}(-B)$  are supercharges (7) with the opposite directed magnetic field.

#### 4. The algebra of SUSY

The supercharges  $\tilde{Q}_{\pm}$  are the integrals of motion of  $H$  and they fulfil the superalgebra

$$\{\tilde{Q}_+, \tilde{Q}_-\} = \tilde{H} \quad (\tilde{Q}_{\pm})^2 = 0 \quad (33)$$

where

$$\tilde{H} = \frac{1}{2} \begin{pmatrix} \tilde{\pi}_- \tilde{\pi}_+ & 0 \\ 0 & \tilde{\pi}_+ \tilde{\pi}_- \end{pmatrix}$$

is also the integral of motion.  $\tilde{H}$  can be treated as a new Hamiltonian and is a time-dependent extension of the usual Hamiltonian  $H$ .

One more integral of motion results from the axial symmetry. It is a  $z$ -component of the angular momentum

$$L_z = z \frac{\partial}{\partial z} - z^* \frac{\partial}{\partial z^*} \quad (34)$$

which satisfies the commutation relation

$$[\tilde{\pi}_{\pm}, L_z] = \mp \tilde{\pi}_{\pm}. \quad (35)$$

It is obvious that  $S_z = \sigma_z/2$  is also an integral of motion. The algebra of SUSY can be extended by  $L_z$  and  $S_z$  which satisfy the following relations

$$\begin{aligned}[\tilde{Q}_{\pm}, L_z] &= \pm \tilde{Q}_{\pm} & [\tilde{Q}_{\pm}, S_z] &= \mp \tilde{Q}_{\pm} \\ [L_z, \tilde{H}] &= [S_z, \tilde{H}] = [S_z, L_z] = 0.\end{aligned}\quad (36)$$

The total moment  $J = L_z + S_z$  commutes with all the generators of the algebra.

Note, that using (19) and (26) we obtain

$$[\tilde{\pi}_-, \tilde{\pi}_+] = i \left( f \frac{\partial f^*}{\partial t} - f^* \frac{\partial f}{\partial t} \right) \quad (37)$$

where  $f$  and  $f^*$  are two linear-independent solutions of equation (27). The Wronskian of equation (27) is constant. We choose to scale the solutions so that the Wronskian is

$$f \frac{\partial f^*}{\partial t} - f^* \frac{\partial f}{\partial t} = -2i \quad (38)$$

then

$$[\tilde{\pi}_-, \tilde{\pi}_+] = 2.$$

Let us introduce the operators of creation and annihilation

$$\tilde{b}_{\pm} = \frac{\tilde{\pi}_{\pm}}{\sqrt{2}} \quad (39)$$

satisfying the commutation relation  $[\tilde{b}_-, \tilde{b}_+] = 1$ . It is convenient to introduce the operators of creation and annihilation  $b_{\pm}$  which are the integrals of motion. Using (22) and (24) we have

$$b_{\pm} = e^{\pm i2\Omega(t)} \tilde{b}_{\pm}. \quad (40)$$

It is obvious that

$$[b_-, b_+] = 1 \quad (41)$$

and using (35)

$$[b_{\pm}, L_z] = \mp b_{\pm}. \quad (42)$$

The new Hamiltonian  $\tilde{H}$  can be written as follows

$$\tilde{H} = \tilde{b}_+ \tilde{b}_- + \sigma_+ \sigma_- = \tilde{b}_+ \tilde{b}_- + s_z + \frac{1}{2} = b_+ b_- + s_z + \frac{1}{2}. \quad (43)$$

In the new notations the supercharges are

$$\tilde{Q}_+ = \tilde{b}_- \sigma_+ = e^{i2\Omega(t)} b_- \sigma_+ \quad \tilde{Q}_- = \tilde{b}_+ \sigma_- = e^{-i2\Omega(t)} b_+ \sigma_-. \quad (44)$$

Let us emphasize that the dependence of the supercharges on the time is not only in phase factor since  $b_{\pm}$  are also the functions of time (see (40), (39) and (28)).

### 5. Eigensolutions of $\tilde{H}$ and a representation of the algebra

Let us choose the orthogonal basis  $|n, m, s\rangle$ , where  $n = 0, 1, 2, \dots$  are the eigenvalues of  $b_+ b_-$ ,  $m$  is the eigenvalue of  $L_z$ ,  $s = \pm \frac{1}{2}$  is the eigenvalue of  $S_z$ .

The action of the operators on the basis is the following

$$\begin{aligned} \tilde{H}|n, m, s\rangle &= (n + s + \frac{1}{2})|n, m, s\rangle \\ L_z|n, m, s\rangle &= m|n, m, s\rangle \\ S_z|n, m, s\rangle &= s|n, m, s\rangle \\ b_-|n, m, s\rangle &= \sqrt{n}|n-1, m-1, s\rangle \\ b_+|n, m, s\rangle &= \sqrt{n+1}|n+1, m+1, s\rangle \\ \tilde{Q}_+|n, m, s\rangle &= \delta_{-1/2, s} e^{i2\Omega(t)} \sqrt{n}|n-1, m-1, \frac{1}{2}\rangle \\ \tilde{Q}_-|n, m, s\rangle &= \delta_{1/2, s} e^{-i2\Omega(t)} \sqrt{n+1}|n+1, m+1, -\frac{1}{2}\rangle. \end{aligned} \quad (45)$$

Note that, in contrast to the stationary case, the basic functions in these relations are time-dependent functions. The considered basis can be obtained by applying the operator of creation  $b_+$  to the ground state

$$|n, m, s\rangle = \frac{1}{\sqrt{n!}} (b_+)^n |0, m-n, s\rangle. \quad (46)$$

The ground state is defined by the equation

$$b_- |0, m, s\rangle = 0. \quad (47)$$

The solution of this equation taking into account spin variables is the following

$$|0, m, s\rangle = C_s(t) \chi_s(z^*)^{-m} \exp \left[ \frac{i}{4} \left( eD(t) + \frac{1}{f(t)} \frac{\partial f(t)}{\partial t} \right) |z|^2 \right] \quad (48)$$

where

$$\chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



are the eigenstates of the spin operator  $S_z$ , from the condition that the wavefunction does not have a pole at zero  $m \leq 0$ ,  $C_s(t)$  is a certain function of time. We may choose this function in the way that eigenstates of  $\tilde{H}$  satisfy the nonstationary Pauli equation. It is easy to check that the wavefunction (48) with

$$C_s(t) = C(f(t))^{m-1} e^{i(m+2s)\Omega(t)} \quad (49)$$

indeed satisfies the nonstationary Pauli equation. Here  $C$  is a constant of normalization of wavefunction (48).

It is known that applying an integral of motion to the solution of wave equation yields a function that itself is the solution of the same wave equation (see for example [11, 14]). Thus, because  $b_+$  is the integral of motion, the wavefunction  $|n, m, s\rangle$  calculated by (46) is also a solution of the nonstationary Pauli equation. For more details on constructing the exact solution of the nonstationary Schrödinger equation with the help of the dynamical symmetry see [14]. Note also that  $\tilde{H}$  has equidistant two-fold degenerated eigenvalues connected with SUSY, except for the unique ground state.

## 6. Conclusion

Originally, the SUSY of the Pauli equation was established in the case of the stationary magnetic field without the electric one. The supercharges in this case do not depend on time. In [11] it was shown that SUSY can also be introduced in the case of the nonstationary magnetic field.

In this paper the SUSY of the Pauli equation is introduced in the case of both the nonstationary magnetic and electric fields. Namely, the system considered is an isotropic two-dimensional nonstationary harmonic charged oscillator in the time-varying uniform magnetic field. The supercharges are obtained as integrals of motion of the Hamiltonian. It is worth noting that although the supercharges  $\tilde{Q}_\pm$  and the new Hamiltonian  $\tilde{H}$  are the integrals of motion, there is implicit time dependence in them. In the case of the stationary magnetic and electric fields the obtained supercharges remain implicitly time dependent (see (31)). Only in the case of the stationary magnetic field without the electric one there exist the supercharges which do not contain time in implicit form. Note also that the obtained algebra can be used for generating solutions of the nonstationary Pauli equation as is shown in section 5.

## References

- [1] Witten E 1981 *Nucl. Phys. B* **185** 513
- [2] Nicolai H 1974 *J. Phys. B: At. Mol. Phys.* **70** 39
- [3] Gendenshteyn L E and Krive N V 1985 *Usp. Fiz. Nauk* **146** 553
- [4] Cooper F, Khare A and Sukhatme U 1995 *Phys. Rep.* **251** 267
- [5] Junker G 1996 *Supersymmetric Methods in Quantum and Statistical Physics* (Berlin: Springer)
- [6] de Grumbrughe M and Rittenberg V 1983 *Ann. Phys.* **151** 99
- [7] Jackiw R 1984 *Phys. Rev. D* **29** 2375
- [8] Khare A and Maharana J 1984 *Nucl. Phys. B* **244** 409
- [9] Gendenshteyn L E 1984 *Zh. Eksp. Teor. Fiz. Pis'ma Red.* **39** 234  
Gendenshteyn L E 1985 *Yad. Fiz.* **41** 261
- [10] Tkachuk V M and Vakarchuk S I 1997 *Phys. Lett. A* **228** 141
- [11] Kostelecky V A, Man'ko V I, Nieto M M and Truax D R 1993 *Phys. Rev. A* **48** 951
- [12] Bagrov V G and Samsonov B F 1996 *Phys. Lett. A* **210** 60
- [13] Tkachuk V M 1997 *J. Phys. Stud. (Lviv)* **1** 314 (in Ukrainian)
- [14] Malkin I A and Man'ko V I 1979 *Dynamical Symmetry and Coherent States of Quantum Systems* (Moskow: Nauka) (in Russian)